MEASURES OF CENTRAL TENDENCY - ARITHMETIC MEAN

SIGNIFICANCE

Quantify A single number is employed as the measure of central tendency to represent a whole set of data.

OBJECTIVES AND FUNCTIONS OF AVERAGES

1. To summarize vast amounts of data: It can be challenging to understand a lot of numerical figures. Averages condense this type of data into a single figure, which facilitates comprehension and retention.

2. To Facilitate Comparison: Because they condense a large amount of statistical data into a single figure, averages are a great tool for conducting comparison research. One can make this kind of comparison over time or at a specific point in time.

3. To Aid in Decision-Making: The average yields these numbers, which serve as a decision-maker's guide. The majority of research and planning decisions are made using average values for certain variables.

4. To understand the cosmos through a sample: Using sample data, averages can also be used to get a sense of the entire universe. An accurate representation of the population average can be obtained by examining the sample average.

5. To trace exact relationships: When quantitative relationships between many groups are to be established, averages become crucial.

6. Base for calculating other measures: In many other stages of statistical analysis, averages are used to calculate a range of other measures, such as kurtosis, skewness, and dispersion.

NEEDS A CENTRAL TENDENCY MEASUREMENT

The following qualities must be present in a good average measure:

1. Strictly defined: To avoid ambiguity and allow for a single, unchanging meaning, an average must be precise and unbending.

- There ought to be no opportunity for using discretion.
- Ideally, it should be specified by an algebraic formula such that any person computing the average given a set of data will always get the same result.

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2. Based on all observations: When calculating the average, each and every item in the series should be taken into account. It cannot be deemed to be typical of the entire group if it is not based on all of the things.
3. It should be least impacted by sampling fluctuations: Averaging should have sampling stability.

- The values so acquired from separate samples should not differ significantly from one another if we pick two or more independent random samples of the same size from a particular population and compute the average for each.

4. Capable of Additional Algebraic Treatment: To increase its usefulness, the average should be able to withstand additional mathematical and statistical examination.
5. Easy to comprehend and calculate: To determine the value of an average, a straightforward approach should be used without sacrificing accuracy or other benefits.
6. Not Much Affected by Extreme Values: An average's value shouldn't be significantly impacted by extreme values. The average number might not accurately reflect the features of the complete set of data if one or two extremely little or large elements have a significant impact on it.

ARITHMETIC MEAN MEANING

The definition of the arithmetic mean is the total of all observation values divided by the total number of observations.

- Most people just refer to it as "Mean" or "Average."
- $\bar{X}$ is often used to indicate it.

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PERSONAL SERIES

As was previously mentioned, an individual series is one in which all of the items are listed separately—that is, each item is assigned a unique value. This implies that there are no item frequencies in individual series.

1. Straightforward Approach

This method involves adding up all the units, dividing the total by the number of items, and using the quotient to get the arithmetic mean.

2. Quick-Cut Approach

Any number is taken as the mean in this procedure, and deviations are computed from this mean.

- A large number of observations or difficulty computing the arithmetic mean using a direct method necessitate the use of the short-cut method.
- The 'Assumed Mean Method' is another name for this technique.

3. STEP DEVIATION TECHNIQUE

The shortcut approach is made even simpler by the Step Deviation Method. Step deviations are obtained by dividing departures from the assumed mean by a common factor (C) in this procedure. The arithmetic mean's value is then determined using these step deviations.

DIFFERENT SERIES

When dealing with discrete series, which are often referred to as frequency arrays or ungrouped frequency distributions, the values of the variables exhibit repeats, meaning that different values of the variables correspond to distinct frequencies. For a discrete series,
N = Sum total of frequency = Σf is the total number of observations.

1. Straightforward Approach

The direct technique divides the entire number of frequencies (2f) by the sum of products (ΣfX), which is obtained by multiplying different items (X) by their corresponding frequencies (f). This yields the basic arithmetic mean. i.e.

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

2. Quick-Cut Approach

The mean in a discrete series can also be determined using the shortcut technique. When computing mean, this method saves a significant amount of time.

3. Step-by-Step Deviation Technique

This approach is a more straightforward version of the shortcut approach. To make calculations easier, the values of the deviations (d) in this approach are divided by a common factor (C). Step deviations are the deviations that remain after this division.

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ANNUAL SERIES

A variable's value is grouped with its corresponding frequencies in different class-intervals (10–20, 20–30, etc.) when it comes to continuous series (grouped frequency distribution).

1. Straightforward Approach

The direct method for continuous series is the same as for discrete series, with the exception that we take the midpoints of each class interval to first turn the continuous series into a discrete series.

2. Quick-Cut Approach

When calculating the mean of a continuous series, the shortcut approach saves a significant amount of time.

3. Step Deviation Technique

The step-deviation approach can further simplify the short-cut method when all of the class-intervals in a continuous series have the same magnitude (width).

ARITHMETIC MEAN IN CERTAIN SITUATIONS

1. 'Less than' or 'More than' cumulative series: After converting the cumulative frequency into a straightforward frequency distribution, compute the mean using the standard procedure.

2. Midpoints are provided: Without translating the mid-values into class intervals, compute the mean using the standard method.

3. Inclusive Class-Intervals: Without transforming the series into an exclusive class-interval series, compute the mean as per standard procedure.

4. Open-end Series: Depending on the structure of the class-intervals of other classes, missing class limits are assumed in order to compute the mean.

5. Unequal Class-Intervals: After computing the mid-values of each interval, the mean can be ascertained in the customary way.

ARITHMETICAL MEAN PROPERTIES

1. The sum of the observations' deviations from their arithmetic mean, or Σ(X – X) = 0, is always zero. That occurs as a result of the arithmetic mean being a point of balance, where the total of the positive and negative deviations from the mean equals one another.

2. $\Sigma(X - \bar{X})^2$ is minimal, which is the sum of the squares of the deviations of the items from their arithmetic mean.

3. Mean of the combined series: We may calculate the combined means of the series as a whole if we know the arithmetic mean and the number of items in two or more linked groups.

4. The arithmetic mean of the new series increases or decreases by k if every observation in the series experiences an increase or reduction. Therefore, $\bar{X} - k$ is the new mean.

5. The mean of these observations is multiplied or divided by the constant if all the items in the series are multiplied or divided by it.

6. If any values can be determined from the arithmetic mean (X), number of items (N), and total of the values (#X), then the third value may be determined with ease.

COMBINED MEAN

The combined mean of two or more distributions with the same number of items and arithmetic means can be found using the following formula:

$$\bar{X}_{1,2} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

EDITED MEAN

Occasionally, when calculating the arithmetic mean, certain incorrect elements may be taken because of an error or oversight. In this scenario, we may determine the proper arithmetic mean without first determining the arithmetic mean.

Correct $\bar{X} = \frac{\Sigma X\ (Wrong) + (\ Correct\ Value\ ) - (\ Incorrect\ Value\ )}{N}$

MEANS WEIGHTED

Meaning: The term "weighted mean" describes the average obtained by assigning varying weights to distinct series components based on their relative importance.

ABITMEAN'S BENEFITS AND DEBTORS

Advantages of the Arithmetic Mean

Due to its advantages, the arithmetic mean is the most commonly employed central tendency measure in practical applications.

1. Easy to Understand and Compute: Basic addition, multiplication, and division skills are necessary for the computation of the arithmetic mean.

- It follows that even someone with rudimentary knowledge may compute the arithmetic mean.
- The definition of the arithmetic mean, such as the value per item or cost per unit, etc., is also easily understood.

2. Confidence: An algebraic formula firmly establishes the arithmetic mean. As a result, the average computation yields the same result for each person. There is no room for intentional prejudice or personal bias when using the arithmetic mean.

3. Based on all items: The arithmetic mean considers all possible values. As a result, it is thought to be more distributionally representative.

4. Least impacted by sample fluctuations: The arithmetic mean is the average that is least impacted by sampling fluctuations.

- The arithmetic mean offers a useful foundation for comparison when a series has a high number of items because abnormalities (errors) in one direction are offset by anomalies in another.

- The arithmetic average is thought to be a stable metric as a result.

5. Easy Method of Comparison: When comparing two or more distributions, the arithmetic average provides a handy way to do it.

6. Algebraic approach: Additional algebraic treatment of the arithmetic mean is possible. Since it can be handled analytically, it is frequently utilized in the calculation of numerous other statistical measures, including mean deviation and standard deviation.

7. No item organization necessary: There is no item arrangement or grouping involved in the computation of the arithmetic mean.

ADVANTAGES OF THE ARITHMETIC MEAN

The arithmetic mean has several limitations and should only be used sparingly, even though it meets the majority of the criteria for a perfect average. Among the arithmetic mean's drawbacks are:

1. Affected by extreme values: Because the arithmetic average is determined using all of the series' items, extreme values (i.e., extremely small or extremely large items) have an excessive impact on it.

2. Assumption in Open-End Classes: When open-end classes are involved, an arithmetic mean is assumed with respect to the open-end classes' magnitude intervals.

3. Nonsensical findings: The arithmetic mean can occasionally produce results that seem almost nonsensical in a given community. For example, it is evident that if a household has 3.2 children on average, the average result is absurd because children cannot be divided into fractions.

4. Not possible in the case of qualitative characteristics: When dealing with qualitative data, such as information on IQ, integrity, smoking habit, etc., the arithmetic mean cannot be calculated. The only appropriate average in these circumstances is the median, which is covered later.

5. Greater emphasis on higher value items: The arithmetic mean has an upward bias, meaning that it places greater weight on higher value items in a series than on lower value items.

- The presence of a large item will significantly raise the average if there are five things total—four little and one large.
- However, the opposite is untrue. The arithmetic average won't decrease significantly if there are five items in the series, four of which have large values and one has a little value.

6. Complete data necessary: Without all of the items in a series, the arithmetic mean cannot be determined.

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7. Calculation by observation is not possible: Like median or mode, arithmetic mean cannot be calculated by only looking at the series.

8. No Graph Use: A graph cannot be used to determine the arithmetic mean.

9. Non-existent value as mean: The arithmetic mean can occasionally be a made-up number that doesn't exist in the series. 16, is the arithmetic mean of 8, 14, 17, and 25. There are no series items with a value of 16.

FORMULAE AT A GLANCE

1. SIMPLE MEAN	**FORMULAE**	**DISCRIPTION**
INDIVIDUALSERIES		
Direct Method	$\bar{X} = \frac{\Sigma X}{N}$	$\bar{X}$ = Arithmetic Mean ΣX = Summation of values of variable X N = Number of observations
Short-cut Method	$\bar{X} = A + \frac{\Sigma d}{N}$	A = Assumed Mean Σd = Sum of deviations of variables from assumed mean
Step Deviation Method	$\bar{X} = A + \frac{\Sigma d'}{N} \times C$	$\Sigma d'$ = Sum of step deviation C = Common Factor
DISCRETE SERIES		
Direct Method	$\bar{X} = \frac{\Sigma fX}{\Sigma f}$	ΣfX = Sum of product of variable (X) and frequencies (f) Σf = Total of Frequencies
Short-cut Method	$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$	Σfd = Sum of Product of Deviations (d) and Respective Frequencies (f)
Step Deviation Method	$\bar{X} = A + \frac{\Sigma fd'}{N} \times C$	$\Sigma fd'$ = sum of product of step deviations (d') and respective frequencies (f)

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CONTINOUS SERIES		
Direct Method	$\bar{X} = \frac{\Sigma fm}{\Sigma f}$	m= Mid points Σfm = sum of product of mid-points (m) and frequencies (f)
Short cut method	$\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$	Σfd = sum of product of deviations (d) from mid-points with the respective frequencies (f)
Step Deviation Method	$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$	$\Sigma fd'$ = sum of product of step deviations (d') and respective frequencies (f)
2. COMBINED MEAN	$\bar{X}_{1,2} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$	$\bar{X}_{1,2}$ = Combined Mean $\bar{X}_1$= Arithmetic Mean of First distribution $\bar{X}_2$ = Arithmetic Mean of second distribution N_1= number of items of first distribution N_2= number of items of second distribution
3. WEIGHTED MEAN	$\bar{X}_W = \frac{\Sigma WX}{\Sigma W}$	$\bar{X}_W$ = Weighted mean ΣWX = sum of product of items and respective weights ΣW = sum of the weights

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